Zadanie 1

a)

$$A_{sA} = \frac{d^2\sqrt{3}}{4} + 3 \cdot \frac{1}{2} \cdot \frac{\pi \cdot d^2}{4} = \frac{20^2\sqrt{3}}{4} + 3 \cdot \frac{1}{2} \cdot \frac{\pi \cdot 20^2}{4} = 644.44 cm^2$$

$$W_{sA} = 2 \cdot A_{sA} \cdot \delta_{min} = 2 \cdot 644.44 \cdot 1 = 1288.88 cm^3$$

$$\oint \frac{ds}{s} = \frac{3 \cdot \frac{1}{2} \cdot \pi \cdot d}{\delta} = \frac{3 \cdot \frac{1}{2} \cdot \pi \cdot 20}{1} = 94.25\ [-]$$

$$I_{sA} = \frac{4A_{sA}^2}{\oint \frac{ds}{s}} = \frac{4 \cdot 644.44^2}{94.25} = 17625.59 cm^4$$

$$G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.25)} = 80 GPa$$

$$\tau_A = \frac{M_s}{W_{sA}} = \frac{3000}{1288.88} = 2.3276 \frac{kN}{cm^2} = 23.276 MPa$$

$$\varphi_A = \frac{M_s L}{G I_{sA}} = \frac{3000 \cdot 1000}{8000 \cdot 17625.59} = 0.02128 rad \approx 1.22°$$

b)

$$\text{a} = \frac{\oint ds}{4} = \frac{3 \cdot \frac{1}{2} \cdot \pi \cdot 20}{4} = 23.562 cm$$

$$A_{sB} = a^2 = 23.562^2 = 555.17 cm^2$$

$$W_{sB} = 2 \cdot A_{sB} \cdot \delta_{min} = 2 \cdot 555.17 \cdot 1 = 1110.34 cm^3$$

$$I_{sB} = \frac{4A_{sA}^2}{\oint \frac{ds}{s}} = \frac{4 \cdot 555.17^2}{94.25} = 13080.69 cm^4$$

$$n_\tau = \frac{\tau_B}{\tau_A} = \frac{\frac{M_s}{W_{sB}}}{\frac{M_s}{W_{sA}}} = \frac{W_{sA}}{W_{sB}} = \frac{1288.88}{1110.34} = 1.161\ [-]$$

$$n_\varphi = \frac{\varphi_B}{\varphi_A} = \frac{\frac{M_s L}{G I_{sB}}}{\frac{M_s L}{G I_{sA}}} = \frac{I_{sA}}{I_{sB}} = \frac{17625.59}{13080.69} = 1.347\ [-]$$

Zadanie 2

a)

$A_s = bh = 18 \cdot 24 = 432cm^2$

$W_{s(z)} = 2 \cdot A_s \cdot \delta_{min} = 2 \cdot 432 \cdot 1 = 864cm^3$

$\oint \frac{ds}{s} = 2 \cdot \frac{18}{1.5} + 2 \cdot \frac{24}{1.0} = 72\ [-]$

$I_{s(z)} = \frac{4A_s^2}{\oint \frac{ds}{s}} = \frac{4 \cdot 432^2}{72} = 10368cm^4$

$\tau_{max}^{(z)} = \frac{M_s}{W_{s(z)}} = \frac{2000}{864} = 2.31 \frac{kN}{cm^2} = 23.1MPa$

$\frac{\varphi_{(z)}}{L} = \frac{M_s}{GI_{s(z)}} = \frac{2000}{8000 \cdot 10368} = 0.0000241 \frac{rad}{cm} = 0.00241 \frac{rad}{m}$

b)

$W_{s(o)} = \frac{1}{3} \cdot \frac{1}{\delta_{max}} \cdot \sum h_i \delta_i^3 = \frac{1}{3} \cdot \frac{1}{1.5} \cdot [2 \cdot 18 \cdot 1.5^3 + 2 \cdot 24 \cdot 1.0^3] = 37.67cm^3$

$n = \frac{\tau_{max}^{(o)}}{\tau_{max}^{(z)}} = \frac{\frac{M_s}{W_{s(o)}}}{\frac{M_s}{W_{s(z)}}} = \frac{W_{s(z)}}{W_{s(o)}} = \frac{864}{37.67} = 22.94\ [-]$

Maksymalne naprężenia styczne wzrosną niemal 23-krotnie.