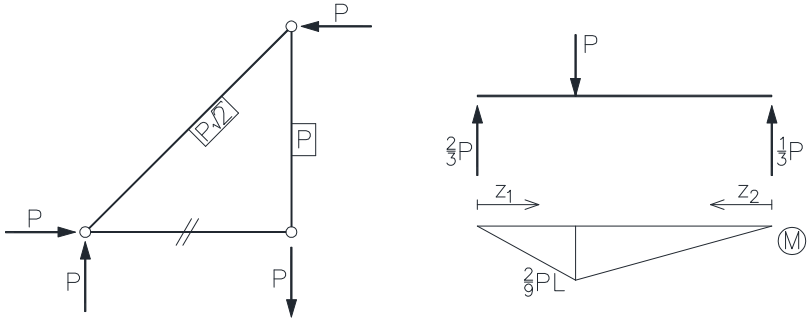
Zadanie 1

Wartości reakcji oraz charakterystycznych sił wewnętrznych dla każdego z układów:



Dla kratownicy:

$$E_P^N = \frac{1}{2}\sum_{k=1}^{n}\frac{N_k^2 l_k}{EA_k} = \frac{1}{2}\left(\frac{P^2 L}{EA} + \frac{\left(P\sqrt{2}\right)^2 L\sqrt{2}}{EA} + \frac{0^2 L}{EA}\right) = \frac{P^2 L}{2EA}\left(1+2\sqrt{2}\right)$$

$$\delta_k = \frac{\partial E_P^N}{\partial P} = \left(\frac{P^2}{2EA}\left(1+2\sqrt{2}\right)\right)' = \frac{PL}{EA}\left(1+2\sqrt{2}\right)$$

Dla belki:

$$E_P^M = \frac{1}{2}\int_0^l \frac{M_x^2}{EI}\,dz$$

Funkcje momentów są różne w różnych przedziałach:

$$M_{x1}(z_1) = \frac{2}{3}Pz_1$$

$$M_{x2}(z_2) = \frac{1}{3}Pz_2$$

$$E_P^M = \frac{1}{2}\int_0^{\frac{1}{3}L}\frac{\left(\frac{2}{3}Pz_1\right)^2}{EI}dz_1 + \frac{1}{2}\int_0^{\frac{2}{3}L}\frac{\left(\frac{1}{3}Pz_2\right)^2}{EI}dz_2 = \frac{P^2}{2EI}\left[\frac{4}{9}\int_0^{\frac{1}{3}L} z_1^2\,dz_1 + \frac{1}{9}\int_0^{\frac{2}{3}L} z_2^2\,dz_2\right] = \frac{P^2}{2EI}\left[\frac{4}{9}\frac{z_1^3}{3}\Big|_0^{\frac{1}{3}L} + \frac{1}{9}\frac{z_2^3}{3}\Big|_0^{\frac{2}{3}L}\right] =$$

$$= \frac{P^2}{2EI}\left[\frac{4}{27}\left(\frac{1}{3}L\right)^3 + \frac{1}{27}\left(\frac{2}{3}L\right)^3\right] = \frac{P^2}{2EI}\left[\frac{4}{27}\frac{L^3}{27} + \frac{1}{27}\frac{8L^3}{27}\right] = \frac{P^2}{2EI}\frac{12L^3}{729} = \frac{2P^2L^3}{243EI}$$

$$\delta_b = \frac{\partial E_P^M}{\partial P} = \left(\frac{2P^2L^3}{243}\right)' = \frac{4PL^3}{243EI}$$